Moving breathing pulses in the one-dimensional complex cubic-quintic Ginzburg-Landau equation

Pablo Gutiérrez,[1,2] Daniel Escaff,[2] Santiago Pérez-Oyarzún,[3] and Orazio Descalzi[2]
[1]*Departamento de Física, FCFM, Universidad de Chile, Casilla 487-3, Santiago, Chile*
[2]*Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes, Avenida San Carlos de Apoquindo 2200 Santiago, Chile*
[3]*FCFM, Universidad Central de Chile, Santa Isabel 1186, Santiago, Chile*

(Received 19 May 2009; revised manuscript received 15 August 2009; published 30 September 2009)

We show and characterize numerically moving breathing pulses in the one-dimensional complex cubic-quintic Ginzburg-Landau equation. This class of stable moving breathing pulses has not been described before for this prototype envelope equation as it arises near the weakly hysteretic onset of traveling waves.

DOI: 10.1103/PhysRevE.80.037202 PACS number(s): 82.40.Bj, 47.20.Ky, 05.70.Ln

In extended systems forced to be out of equilibrium, by moving only one control parameter, the primary instability can be stationary, oscillatory, or lead to a wavy structure [1]. As the control parameter is increased the system may go through a secondary instability [2] and lastly exhibit spatiotemporal chaos. At the onset of a primary instability the dynamics of a system can be captured by considering a weakly nonlinear approach. The results are amplitude equations for the critical modes, which are universal in the sense that they depend only on the type of instability and the symmetries of the problem, so that physical systems of different nature may exhibit analogous behaviors. For one-dimensional systems, assuming translational ($x \to x+\alpha$) and parity ($x \to -x$) symmetries (the time inversion $t \to -t$ is not a symmetry because of the dissipation), generic amplitude equations result in (supercritical) complex cubic Ginzburg-Landau (GL) equations, which have been studied for over three decades [3]. In one-dimensional systems apart from traveling waves and fronts the complex cubic GL equation accepts sinks, sources, and hole solutions, as it was showed by Nozaki and Bekki [4], but never stable pulses.

A mixture of binary fluids, heated from below, may exhibit coexistence between conductive and convective states giving rise to stable localized structures such as pulses [5]. Numerical simulations [6] and experiments [7] show that large scale mean concentration current loops influence the localized traveling wave, so that localized pulses in binary-fluid mixtures are not weakly nonlinear structures. Nevertheless, at least formally, a system undergoing an oscillatory instability with a finite wavelength and allowing localized structures can be described, close to the instability, by coupled complex cubic-quintic GL equations for counter-propagating waves. Recently, this theoretical approach has been successfully used to understand a universal mechanism explaining partial annihilation of colliding dissipative pulses [8]. Thus, we consider complex cubic-quintic GL equations as useful phenomenological dispersive-dissipative models supporting stable localized structures. When pulses and holes are not interacting with each other they satisfy (in the moving frame) one-dimensional (subcritical) single complex cubic-quintic GL equations [9,10],

$$\partial_t A = \mu A + \beta |A|^2 A + \gamma |A|^4 A + D \partial_{xx} A. \tag{1}$$

The subscript x denotes partial derivative with respect to x, $A(x,t)$ is a complex field, and the parameters $\beta = \beta_r + i\beta_i$, $\gamma = \gamma_r + i\gamma_i$, and $D = D_r + iD_i$ are in general complex. The signs of the parameters $\beta_r > 0$ and $\gamma_r < 0$ are chosen in order to guarantee that the bifurcation is subcritical and saturates to quintic order. The control parameter μ is taken to be real without loss of generality. Thual and Fauve [9] showed, two decades ago, the existence of stable stationary pulses as long as van Saarloos and Hohenberg [10] pointed out the fact (from numerical observations) that stable stationary pulses exist in a narrow range where there is a coexistence between zero and the nonzero homogeneous solutions (traveling waves). Coexistence of the trivial and the homogeneous solution is not a sufficient condition to have stable localization in Eq. (1). Indeed, Descalzi *et al.* [11] reported that the appearance of stationary pulses is related to a saddle-node bifurcation reducing the stability domain in agreement with Saarloos and Hohenberg. Stationary solutions here mean that the amplitude $|A|$ only depends on space but not on time. However, Re A and Im A are functions on space and time.

While hole solutions reported by Nozaki and Bekki [4] for the complex cubic GL equation resulted to be structurally unstable [3], two (structurally) stable stationary hole solutions for the complex cubic-quintic GL equation were reported by Sakaguchi [12]. In addition to the previous mentioned nonmoving fixed shaped solutions Eq. (1) admits pulses, which can exhibit periodic, quasiperiodic, or chaotic breathing motion [13]. More recently Descalzi and Brand showed the existence of nonmoving breathing holes for periodic and Neumann boundary conditions [14,15]. Apart

TABLE I. Reported localized structures in the complex cubic-quintic Ginzburg-Landau equation.

	Pulses	Holes
Nonmoving fixed shaped	Refs. [9,10]	Ref. [12]
Nonmoving breathing	Ref. [13]	Refs. [14,15]
Moving fixed shaped	Ref. [16]	Ref. [14]
Moving breathing	Missing	Refs. [14,15]

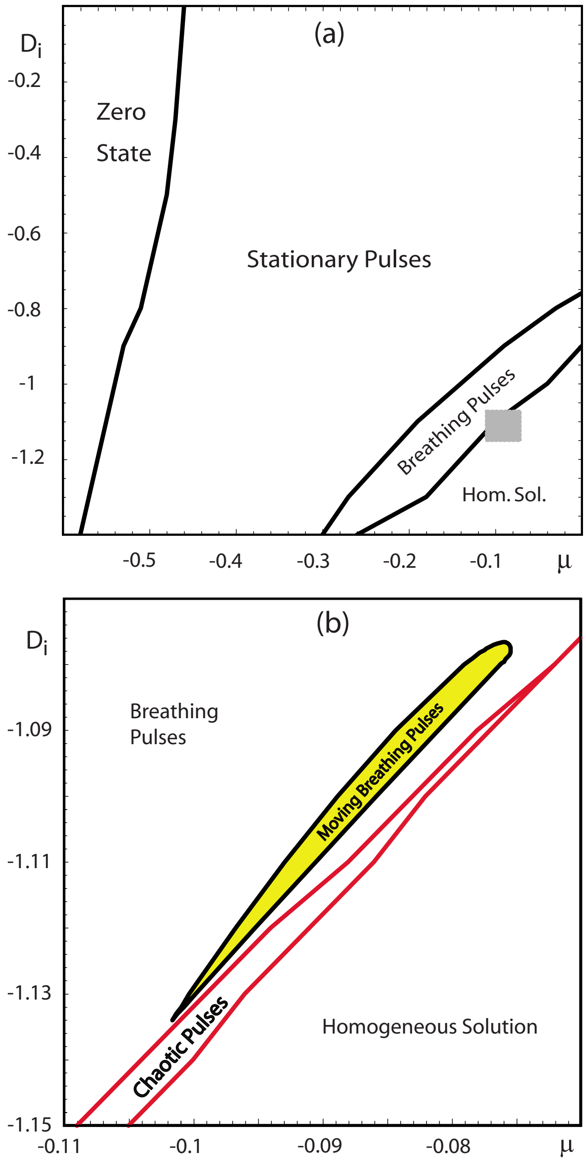

FIG. 1. (Color online) Phase diagram of Eq. (1) in the (μ, D_i) space for the parameters $\beta_r=3$, $\beta_i=1$, $\gamma_r=-2.75$, $\gamma_i=1$, and $D_r=0.9$. (a) The picture shows regions for four different classes of solutions when we start using ICP or ICA: zero states, stationary pulses, nonmoving breathing pulses, and homogeneous solutions. (b) Amplification of the depicted gray square in (a). ICAs lead to chaotic pulses and moving breathing pulses (yellow or gray region) close to the border separating regions of breathing pulses and homogeneous solutions.

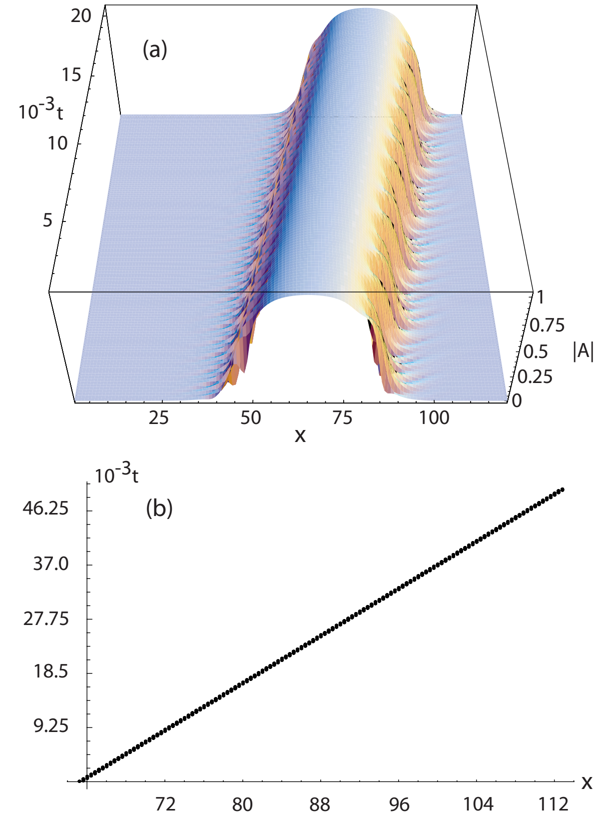

FIG. 2. (Color online) (a) Space-temporal evolution of the amplitude of a moving breathing pulse. (b) Position of the top of the amplitude of the moving breathing pulse as a function of time. The velocity results to be $v=0.001$. The parameters are $\mu=-0.0872$, $\beta_r=3$, $\beta_i=1$, $\gamma_r=-2.75$, $\gamma_i=1$, $D_r=0.9$, and $D_i=-1.1$.

from nonmoving objects the complex cubic-quintic GL equation admits also moving solutions either fixed shaped or breathing. Afanasjev *et al.* [16] reported fixed shaped moving pulses, while fixed shaped moving holes have been studied in Ref. [14]. In both cases the motion is connected to the spatial asymmetry of the solutions. The scenario for moving breathing pulses and holes is different in the sense that while moving breathing holes have been studied, either for periodic or Neumann boundary conditions [14,15], the corresponding moving breathing pulse is missing. For a compact picture see Table I. We note that the creeping soliton presented by Soto-Crespo *et al.* [17] is related to composite pulses and not to plain pulses. In a different context, namely, reaction-diffusion systems, Mimura *et al.* reported oscillatory traveling pulses with breathing motion [18]. We note that in the one-dimensional complex cubic GL equation, without a stabilizing quintic order term, chaotic, stationary, and time-periodic states corresponding to a periodic array of pulses have been found [19].

The goals of this Brief Report are to show and to study the existence of moving breathing pulses (MBPs) in the one-dimensional complex cubic-quintic Ginzburg-Landau equation. We have performed a numerical study of Eq. (1), using a fourth-order Runge-Kutta method for the time evolution ($dt=0.05$) and finite differences to approximate the spatial derivatives ($dx=0.4$), in a typical domain $L=240$ ($N=600$ points) with periodic boundary conditions. For comparison we also studied smaller and larger box sizes to guarantee that there are no finite size effects for sufficiently large box size. We were performing up to 2×10^6 iterations to check for long transients corresponding to a total time of $T=10^5$. We also changed dx and dt to verify that none of the results presented depends sensitively on the discretization used. The selection of boundary and initial conditions is a very important issue by looking for solutions in the complex cubic-quintic Ginzburg-Landau equation. Notice that Neumann boundary conditions preclude the existence of moving

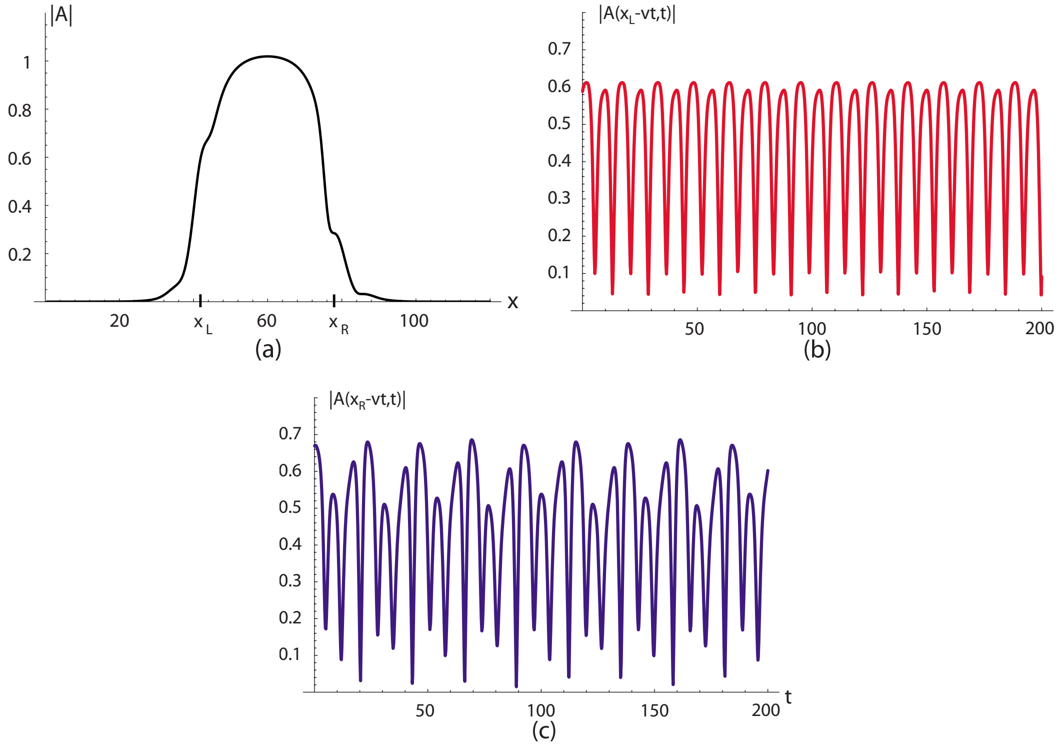

FIG. 3. (Color online) (a) Snapshot of the amplitude of a breathing moving pulse. (b) Time series for the amplitude at $x = x_L$, as a function of time, in the moving frame. (c) Time series for the amplitude at $x = x_R$, as a function of time, in the moving frame. Parameters are the same as in Fig. 2.

objects [15]. In this work we use two classes of initial conditions: initial conditions in phase (ICPs) and initial conditions in antiphase (ICAs). The former is the typical one and is obtained by using Im $A(x)=0$ and localized Re $A(x)$ positive (or negative) and the latter (which may lead to moving objects) by choosing Im $A(x)=0$ and Re $A(x)$ with a positive and a negative part.

Since Eq. (1) involves seven parameters and we can scale the amplitude A, the time t, and the space x, the problem is reduced to a four-dimensional parameter space. Setting the parameters γ and D real and $\beta = \beta_r + i\beta_i$, Eq. (1) still remains nonvariational due to the existence of β_i. Using this approach in Ref. [20] we found zero states (for enough negative control parameter μ), stationary pulses, stationary and moving holes, and moving pulses. On the other hand, negative values of the linear dispersion D_i may lead to breathing solutions as it has been showed in [13,21]. Thus in this Brief Report we use the parameters $\beta_r = 3$, $\beta_i = 1$, $\gamma_r = -2.75$, $\gamma_i = 1$, $D_r = 0.9$, and D_i negative and varied from 0 to -1.4. Figure 1 summarizes our results. In Fig. 1(a) we can see four different classes of solutions when we start using ICPs or ICAs: zero states, which mean that any perturbation decays to zero; stationary pulses; nonmoving breathing pulses; and homogeneous solutions, which are traveling waves whose amplitudes are homogeneous.

Close to the border separating regions of nonmoving breathing pulses and homogeneous solutions we found narrow regions of chaotic pulses and MBPs [see Fig. 1(b)]. Chaotic pulses can be reached using ICP or ICA, while MBPs can only be reached with ICA. Thus there exists coexistence between nonmoving and MBPs. A three-dimensional plot

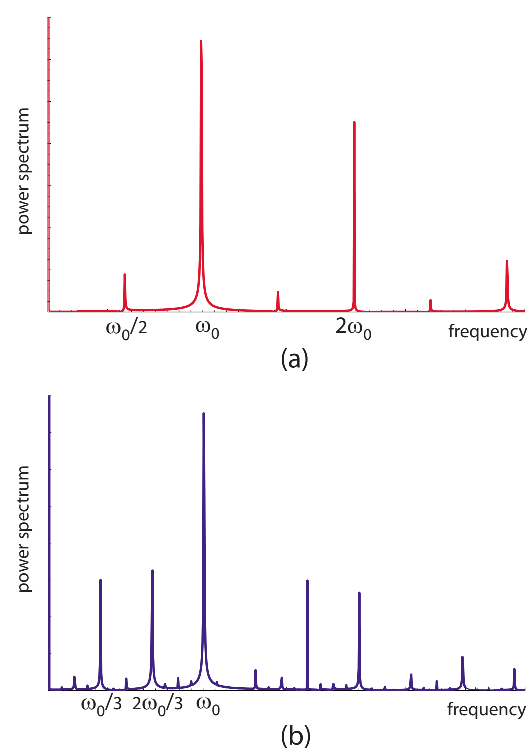

FIG. 4. (Color online) (a) Power spectrum of the time series for the amplitude at $x = x_L$ (in the moving frame). (b) Power spectrum of the time series for the amplitude at $x = x_R$ (in the moving frame). Fundamental frequency $\omega_0 = 0.82$.

showing the space-time dynamics of a moving breathing pulse is brought out in Fig. 2(a). Studying the position of the top of MBP as a function of time we found out that MBPs move with a constant speed [see Fig. 2(b)]. Setting all parameters fixed (including D_i), except μ, we can describe the appearance (or disappearance) of moving breathing pulses by moving the control parameter μ. For D_i fixed and in a certain range of D_i, there exist $\mu=\mu_1$ and $\mu=\mu_2$, so that for $\mu<\mu_1$ we get nonmoving breathing pulses either for ICP or for ICA. Increasing μ and for $\mu_1<\mu<\mu_2$ we can obtain moving breathing pulses by using ICA [see Fig. 1(b)]. We conclude that at $\mu=\mu_1$ ($\mu=\mu_2$) MBPs appear (disappear) through a saddle-node bifurcation. For the case $D_i=-1.1$ the bifurcation parameters are $\mu_1=-0.0888$ and $\mu_2=-0.0858$. Further characterization of MBPs leads us to the analysis of time series for the amplitude at fixed spatial points (in the moving frame), x_L and x_R, which are equidistant from the center of the breathing moving pulse (see Fig. 3). The power spectra of both time series for the amplitudes at $x=x_L$ and $x=x_R$ (in the right moving frame) show a fundamental frequency $\omega_0=0.82$. However, the power spectrum associated to $x=x_L$ shows subharmonic frequencies such as $\omega_0/2$ [see Fig. 4(a)], while the power spectrum associated to $x=x_R$ shows subharmonic frequencies such as $\omega_0/3$ and $2\omega_0/3$ [see Fig. 4(b)]. In the case of fixed shaped moving pulses [16] the motion is connected to the asymmetry of the shape of the pulse. For moving breathing pulses the asymmetry becomes reflected in the differences between both power spectra as signature of spatiotemporal asymmetry.

In summary we have shown the existence of moving breathing pulses in the one-dimensional complex cubic-quintic Ginzburg-Landau equation. Stationary, moving fixed shaped, and nonmoving breathing pulses have been reported before in the literature, but moving breathing pulses were missing. In this Brief Report we have shown that moving breathing pulses coexist with nonmoving breathing pulses and are characterized by a constant speed and asymmetry reflected in different power spectra of the time series for the amplitude at fixed spatial points (in the moving frame), which are equidistant from the center of the breathing moving pulse.

O.D. wishes to acknowledge the support of FONDECYT (Project No. N1070098). O.D., P.G., and D.E. acknowledge additional support by FAI (Universidad de los Andes). S.P.-O. wishes to acknowledge the support of Universidad Central de Chile (Project Estructuras Localizadas Estables en Sistemas Disipativos).

[1] M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[2] M. A. Miranda and J. Burguete, Phys. Rev. E **78**, 046305 (2008).
[3] I. S. Aranson and L. Kramer, Rev. Mod. Phys. **74**, 99 (2002).
[4] K. Nozaki and N. Bekki, J. Phys. Soc. Jpn. **53**, 1581 (1984).
[5] P. Kolodner, D. Bensimon, and C. M. Surko, Phys. Rev. Lett. **60**, 1723 (1988).
[6] W. Barten, M. Lücke, and M. Kamps, Phys. Rev. Lett. **66**, 2621 (1991).
[7] B. L. Winkler and P. Kolodner, J. Fluid Mech. **240**, 31 (1992).
[8] O. Descalzi, J. Cisternas, D. Escaff, and H. R. Brand, Phys. Rev. Lett. **102**, 188302 (2009).
[9] O. Thual and S. Fauve, J. Phys. France **49**, 1829 (1988).
[10] W. van Saarloos and P. C. Hohenberg, Phys. Rev. Lett. **64**, 749 (1990).
[11] O. Descalzi, M. Argentina, and E. Tirapegui, Phys. Rev. E **67**, 015601(R) (2003).
[12] H. Sakaguchi, Prog. Theor. Phys. **86**, 7 (1991).
[13] R. J. Deissler and H. R. Brand, Phys. Rev. Lett. **72**, 478 (1994).
[14] O. Descalzi and H. R. Brand, Phys. Rev. E **72**, 055202(R) (2005).
[15] O. Descalzi and H. R. Brand, Prog. Theor. Phys. **119**, 725 (2008).
[16] V. V. Afanasjev, N. Akhmediev, and J. M. Soto-Crespo, Phys. Rev. E **53**, 1931 (1996).
[17] J. M. Soto-Crespo, N. Akhmediev, and A. Ankiewicz, Phys. Rev. Lett. **85**, 2937 (2000).
[18] M. Mimura, M. Nagayama, H. Ikeda, and T. Ikeda, Hiroshima Math. J. **30**, 221 (2000).
[19] W. Schöpf and L. Kramer, Phys. Rev. Lett. **66**, 2316 (1991).
[20] O. Descalzi, P. Gutiérrez, and E. Tirapegui, Int. J. Mod. Phys. C **16**, 1909 (2005).
[21] O. Descalzi, Physica A **327**, 23 (2003).